



# Cambridge International AS & A Level

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**MATHEMATICS**

**9709/52**

Paper 5 Probability & Statistics 1

**October/November 2023**

MARK SCHEME

Maximum Mark: 50

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |   |
|--------|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent   |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)   |
| CWO    | Correct Working Only  |
| ISW    | Ignore Subsequent Working   |
| SOI    | Seen Or Implied   |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working   |
| AWRT   | Answer Which Rounds To  |

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| Question | Answer   | Marks     | Guidance   |
|----------|--|-----------|--|
| 1(a)     | $p + r + 0.55 = 1$   | <b>M1</b> | Using sum of probabilities = 1 to form an equation.<br>Accept $p + r = 0.45$ oe.   |
|          | $p + 2r + 0.45 = 1.1$  | <b>M1</b> | Use $E(X) = 1.1$ to form an equation.<br>Accept $p + 2r = 0.65$ oe.<br>NB: These marks can be gained in either order; the second M may have an algebraic substitution. |
|          | $p = 0.25, r = 0.2$  | <b>A1</b> | If both Ms not awarded, <b>SC B1</b> for $p = 0.25, r = 0.2$ stated.   |
|          |  | <b>3</b>  |  |
| 1(b)     | $[\text{Var}(X) =] [0.4 \times 0^2 + ] \text{their } 0.25 [ \times 1^2 ] + (\text{their } 0.2) \times 2^2 + 0.15 \times 3^2 - 1.1^2$<br>[=[0+] 0.25 + 0.8 + 1.35 - 1.21] | <b>M1</b> | Correct formula for variance method using their probability distribution table, $0 < \text{their } P(x) < 1$ .   |
|          | $= 1.19, 1 \frac{19}{100}$   | <b>A1</b> | If M0 awarded, <b>SC B1</b> for 1.19 www.<br>$\frac{119}{100}$ is A0.  |
|          |  | <b>2</b>  |  |

| Question | Answer  | Marks     | Guidance   |
|----------|---|-----------|--|
| 2(a)     | <b>Method 1:</b>  |           |  |
|          | [P(5) = 0.2]<br>[P(X < 7) =] $1 - 0.8^6$  | <b>M1</b> | $1 - 0.8^n, n = 6, 7.$   |
|          | = 0.738, $\frac{11529}{15625}$  | <b>A1</b> | 0.737856 to at least 3SF.  |
|          | <b>Method 2:</b>  |           |  |
|          | [P(X < 7) =] $0.2 + 0.2 \times 0.8 + 0.2 \times 0.8^2 + 0.2 \times 0.8^3 + 0.2 \times 0.8^4 + 0.2 \times 0.8^5$ | <b>M1</b> | $0.2 + 0.2 \times 0.8 + 0.2 \times 0.8^2 + 0.2 \times 0.8^3 + 0.2 \times 0.8^4 + 0.2 \times 0.8^5 (+0.2 \times 0.8^6)$ |
|          | = 0.738, $\frac{11529}{15625}$  | <b>A1</b> | 0.737856 to at least 3SF.  |
|          |   | <b>2</b>  |  |

| Question | Answer   | Marks     | Guidance   |
|----------|--|-----------|--|
| 2(b)     | <b>Method 1:</b>   |           |  |
|          | $[P(5, 6, 7) = ]$ ${}^{10}C_5 (0.2)^5 (0.8)^5 + {}^{10}C_6 (0.2)^6 (0.8)^4 + {}^{10}C_7 (0.2)^7 (0.8)^3$   | <b>M1</b> | One term:<br>${}^{10}C_x (p)^x (1-p)^{10-x}$ , $0 < p < 1$ , $x \neq 0, 10$ .      |
|          | $[0.02642 + 5.505 \times 10^{-3} + 7.864 \times 10^{-4}]$  | <b>A1</b> | Correct expression, accept unsimplified, no terms omitted leading to final answer. |
|          | $= 0.0327$   | <b>B1</b> | awrt   |
|          | <b>Method 2:</b>   |           |  |
|          | $[P(X < 8) - P(X \leq 4) = 1 - P(X \geq 8) - P(X \leq 4) =]$ $1 - \{ {}^{10}C_8 (0.2)^8 (0.8)^2 + {}^{10}C_9 (0.2)^9 0.8 + (0.2)^{10} \}$ $- \{ (0.8)^{10} + {}^{10}C_1 (0.2)(0.8)^9 + {}^{10}C_2 (0.2)^2 (0.8)^8 + {}^{10}C_3 (0.2)^3 (0.8)^7 + {}^{10}C_4 (0.2)^4 (0.8)^6 \}$ $[1 - \{ 7.373 \times 10^{-5} + 4.096 \times 10^{-6} + 1.024 \times 10^{-7} \} -$ $\{ 0.1074 + 0.2684 + 0.3020 + 0.2013 + 0.08808 \}]$ | <b>M1</b> | One term:<br>${}^{10}C_x (p)^x (1-p)^{10-x}$ , $0 < p < 1$ , $x \neq 0, 10$ .      |
|          |  | <b>A1</b> | Correct expression, accept unsimplified, no terms omitted leading to final answer. |
|          | $= 0.0327$   | <b>B1</b> | awrt   |
|          |  | <b>3</b>  |  |



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| Question | Answer  | Marks     | Guidance   |
|----------|---|-----------|--|
| 3        | [Mean = $200 \times 0.15 =$ ] 30<br>[Var = $200 \times 0.15 \times 0.85 =$ ] 25.5 | <b>B1</b> | 30 and 25.5, $25 \frac{1}{2}, \frac{51}{2}$ seen, allow unsimplified.<br>May be seen in standardisation formula.<br>$[\sigma =] 5.049 \leq \sigma \leq 5.05[0], \frac{\sqrt{102}}{2}$ implies correct variance.<br>Correct notation is required. |
|          | $[P(X > 40) =] P(Z > \frac{40.5 - 30}{\sqrt{25.5}})$                              | <b>M1</b> | Substituting <i>their</i> mean and <i>their</i> positive 5.04975 into $\pm$ standardisation formula (any number for 40.5), not <i>their</i> $\sigma^2$ or $\sqrt{\text{their } \sigma}$ .  |
|          | $[1 - \Phi(2.079)]$<br>$1 - 0.9812$   | <b>M1</b> | Using continuity correction 39.5 or 40.5 in <i>their</i> standardisation formula.  |
|          | $= 0.0188$  | <b>A1</b> | $0.01875 < p \leq 0.0188$  |
|          |   | <b>5</b>  |  |

| Question  | Answer   | Marks     | Guidance   |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
|-----------|--|-----------|--|------|---------|----|-----|-------|----|---------|---------|----|---------|--|----|---|-----------|---|
| 4(a)      | <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; border-bottom: 1px solid black;">Aces</th> <th style="border-left: 1px solid black; border-bottom: 1px solid black;"></th> <th style="text-align: right; border-bottom: 1px solid black;">Jets</th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">9 8 6 4</td> <td style="border-left: 1px solid black; text-align: center;">16</td> <td style="text-align: left;">6 8</td> </tr> <tr> <td style="text-align: right;">4 3 1</td> <td style="border-left: 1px solid black; text-align: center;">17</td> <td style="text-align: left;">0 4 4 5</td> </tr> <tr> <td style="text-align: right;">2 2 1 0</td> <td style="border-left: 1px solid black; text-align: center;">18</td> <td style="text-align: left;">1 1 8 8</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; text-align: center;">19</td> <td style="text-align: left;">0</td> </tr> </tbody> </table> | Aces      |  | Jets | 9 8 6 4 | 16 | 6 8 | 4 3 1 | 17 | 0 4 4 5 | 2 2 1 0 | 18 | 1 1 8 8 |  | 19 | 0 | <b>B1</b> | <p>Correct stem, ignore extra values (not in reverse, not split).</p> <p>If a split stem-and-leaf plot is used (i.e., stem values are repeated), the remaining B marks are available.</p> |
|           | Aces   |           | Jets   |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
| 9 8 6 4   | 16   | 6 8       |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
| 4 3 1     | 17   | 0 4 4 5   |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
| 2 2 1 0   | 18   | 1 1 8 8   |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
|           | 19   | 0         |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
| <b>B1</b> | <p>Correct Aces labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.</p>  |           |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
|           | <p>Key: 1   17   0 means 171 cm for the Aces and 170 cm for the Jets</p>   | <b>B1</b> | <p>Correct Jets labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.</p>           |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
|           |  | <b>B1</b> | <p>Correct key for their diagram, need both teams labelled and ‘cm’ stated at least once here, or in leaf headings or title.</p> |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |
|           |  | <b>4</b>  |  |      |         |    |     |       |    |         |         |    |         |  |    |   |           |   |

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| Question | Answer   | Marks     | Guidance   |
|----------|--|-----------|--|
| 4(b)     | Median = 173 [cm]  | <b>B1</b> | Accept $Q_2$ ; must be identified.   |
|          | [IQR =] 181 – 168  | <b>M1</b> | $180 \leq UQ \leq 182 - 166 \leq LQ \leq 169$<br>Implied if both quartile values are stated and an appropriate IQR calculated accurately.  |
|          | 13 [cm]  | <b>A1</b> | www<br>If M0 scored <b>SC B1</b> for 13 www.   |
|          |  | <b>3</b>  |  |
| 4(c)     | Jets have a greater variety of heights.<br>Jets have a wider range of height.<br>Jets have a greater/larger/bigger/wider/'more' spread of heights.<br>Aces have a smaller variety of height etc... | <b>B1</b> | [Jets IQR = 18 cm, Range = 24 cm<br>Aces IQR = <i>their</i> <b>4(b)</b> , Range = 18 cm]<br>Comment about spread in context, must include height.<br>Comparison of values does not score until a comment in context is made.<br>If values for range or IQR are stated, they must be correct or FT from <b>4(b)</b> .<br>If more than one comment about spread, mark the final comment.<br>Additional comments about central tendency score B0. |
|          |  | <b>1</b>  |  |

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| Question | Answer  | Marks     | Guidance  |
|----------|---|-----------|---|
| 5(a)(i)  | $[P(X < 170) =] P\left(Z < \frac{170-166}{10}\right)$     | <b>M1</b> | Use of $\pm$ standardisation formula with 170, 166 and 10 substituted appropriately, condone $10^2$ , $\sqrt{10}$ , condone continuity correction $\pm 0.5$ . |
|          | $[= P(Z < 0.4) =] 0.655$                                  | <b>A1</b> | $0.655 \leq p < 0.6555$<br>If M0 awarded, <b>SC B1</b> for correct answer www.  |
|          |   | <b>2</b>  |   |
| 5(a)(ii) | $\left[ P\left(Z > \frac{h-166}{10}\right) = 0.4 \right]$ | <b>B1</b> | $0.253 \leq z \leq 0.2535$ or $-0.2535 \leq z \leq -0.253$ seen.  |
|          | $\frac{h-166}{10} = 0.253$                                | <b>M1</b> | Use of the $\pm$ standardisation formula with $h$ , 166, 10 and a $z$ -value (not $1 - z$ -value), not $10^2$ , $\sqrt{10}$ , no continuity correction.       |
|          | $h = 168.53$  | <b>A1</b> | If M0 scored, <b>SC B1</b> for $168.53 \leq h \leq 168.535$ , 168.5.<br><b>SC B1</b> for 168.54 from $z = 0.254$ .  |
|          |   | <b>3</b>  |   |

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| Question | Answer   | Marks     | Guidance   |
|----------|--|-----------|--|
| 5(b)     | $\left[ P(X > 0) = P\left(Z > \frac{0 - \mu}{\sigma}\right) = \right] P\left(Z > \frac{[0] - \mu}{\frac{2}{3}\mu}\right)$ <p>Or <math>P\left(Z &gt; \frac{[0] - \frac{3}{2}\sigma}{\sigma}\right)</math></p> | <b>M1</b> | Use of the $\pm$ standardisation formula with 0, $\mu$ and $\frac{2}{3}\mu$ substituted for $\sigma$ .<br>Or use of the $\pm$ standardisation formula with 0, $\sigma$ and $\frac{3}{2}\sigma$ substituted for $\mu$ . |
|          | = P(Z > -1.5)  | <b>A1</b> | -1.5 seen, no additional terms (e.g. $x - 1.5$ A0). Condone $Z < 1.5$ .<br>If M0 scored, <b>SC B1</b> $Z > -1.5$ or $Z < 1.5$ seen www.  |
|          | = 0.933 final answer   | <b>A1</b> | $0.933 \leq p < 0.9333$ .<br>If M0 scored, <b>SC B1</b> $0.933 \leq p < 0.9333$ seen www.  |
|          |  | <b>3</b>  |  |

| Question | Answer | Marks  | Guidance   |
|----------|--------|--|--|
| 6(a)     |        | <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> | <p>1st column, 2 branches identified X, Y with probabilities <math>\frac{1}{2}</math>, <math>\frac{1}{2}</math> indicated.</p> <p>2nd column (1st marble pick) of 4 branches identified R B R B (oe) and probabilities <math>\frac{7}{10}, \frac{3}{10}, \frac{4}{5}, \frac{1}{5}</math> indicated appropriately.</p> <p>3rd column (2nd marble pick) of 8 branches identified R B R B R B R [B] (oe) and probabilities <math>\frac{7}{10}, \frac{3}{10}, \frac{8}{10}, \frac{2}{10}, \frac{4}{5}, \frac{1}{5}, 1, [0]</math>.</p> <p>Condone omission of YBB branch if YBR branch is fully correct.</p> <p>Ignore any additional columns of branches.</p> <p>If separate tree diagrams for bags X and Y, BOB1B1 max if bags clearly identified.</p> |
|          |        | 3  |  |

| Question | Answer  | Marks        | Guidance  |
|----------|---|--------------|---|
| 6(b)     | $[P(\text{both same colour}) = P(\text{BB}) + P(\text{RR}) = P(\text{XBB}) + P(\text{XRR}) + P(\text{YRR}) =]$ $\frac{1}{2} \times \frac{3}{10} \times \frac{2}{10} + \frac{1}{2} \times \frac{7}{10} \times \frac{7}{10} + \frac{1}{2} \times \frac{4}{5} \times \frac{4}{5}$ $\left[ = \frac{6}{200} + \frac{49}{200} + \frac{16}{50}, 0.03 + 0.245 + 0.32 \right]$ | <b>B1 FT</b> | $\left[ P(\text{BB}) = \right] \left[ \frac{1}{2} \times \frac{3}{10} \times \frac{2}{10} \left[ + \frac{1}{2} \times \frac{1}{5} \times 0 \right] = \right] \frac{6}{200}$ <p>seen. Accept unsimplified.<br/>FT from 6(a) unsimplified only with 3 term probabilities.</p>   |
|          |   | <b>B1 FT</b> | <p>Either <math>[P(\text{XRR}) =] \frac{1}{2} \times \frac{7}{10} \times \frac{7}{10}</math> or</p> <p><math>[P(\text{YRR}) =] \frac{1}{2} \times \frac{4}{5} \times \frac{4}{5}</math> seen.</p> <p>FT from 6(a) unsimplified only with 3 term probabilities.</p>  |
|          |   | <b>M1</b>    | $[P(\text{BB}) + P(\text{XRR}) + P(\text{YRR}) =]$ <p><i>their</i> <math>\frac{6}{200} + \textit{their} \frac{49}{200} + \textit{their} \frac{16}{50}</math></p> <p>Accept unsimplified, consistent with tree diagram if not clearly identified by notation.</p>  |
|          | $= \frac{119}{200}, 0.595$  | <b>A1</b>    |   |
|          |   | <b>4</b>     | <p>Special case: if ½ omitted consistently in the tree diagram and the calculation (i.e., no probability for picking the bags), no FT.</p> <p><b>SC B1</b> <math>[P(\text{BB}) =] \frac{3}{10} \times \frac{2}{10} \left[ + \frac{1}{5} \times 0 \right]</math></p> <p><b>SC B1</b> <math>[P(\text{RR}) =] \frac{7}{10} \times \frac{7}{10} + \frac{4}{5} \times \frac{4}{5}</math></p> <p><b>SC B1</b> <math>\frac{3}{10} \times \frac{2}{10} \left[ + \frac{1}{5} \times 0 \right] + \frac{7}{10} \times \frac{7}{10} + \frac{4}{5} \times \frac{4}{5}</math></p> |

| Question | Answer   | Marks     | Guidance   |
|----------|--|-----------|--|
| 6(c)     | $P(\text{bag } Y \mid \text{different colours}) = \left( \frac{P(\text{bag } Y \cap \text{different colours})}{P(\text{different colours})} \right)$ $\frac{\frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}{1 - \text{their} \left( \frac{119}{200} \right)} \text{ or } \frac{\frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}{\frac{1}{2} \times \frac{7}{10} \times \frac{3}{10} + \frac{1}{2} \times \frac{3}{10} \times \frac{8}{10} + \frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}$ | <b>M1</b> | FT from <i>their 6(a)</i> and <i>their 6(b)</i> with 3 term probabilities unsimplified only or correct.<br>Accept $\frac{\frac{4}{5} + \frac{1}{5}}{\frac{50}{81} + \frac{10}{81}}$ , $\frac{\frac{2}{25} + \frac{1}{10}}{\frac{0.08 + 0.1}{0.405}}$ .   |
|          | $= \left[ \frac{\frac{9}{50}}{\frac{81}{200}} \right] = \frac{4}{9}, 0.444$  | <b>A1</b> | Accept $\frac{36}{81}$ , 0.4.  |
|          |  | <b>2</b>  | Special case: if $\frac{1}{2}$ omitted consistently in the tree diagram and the calculation (ie no probability for picking the bags), no FT.<br><b>SC B1</b> $\frac{\frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}{1 - \text{their } \mathbf{6(b)}}$ ,<br>or $\frac{\frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}{\frac{7}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{8}{10} + \frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}$ . |



| Question | Answer                              | Marks     | Guidance   |
|----------|-------------------------------------|-----------|--|
| 7(a)     | $\frac{5! \times 4!}{2! \times 2!}$ | <b>M1</b> | $\frac{5! \times 4!}{e}$ , $e$ a positive integer, 1 can be implied. No other terms on numerator. No addition etc. |
|          |                                     | <b>M1</b> | $\frac{f}{2! \times g!}$ , $f$ a positive integer, $g = 1, 2$ . No other terms on denominator.                     |
|          | 720                                 | <b>A1</b> |  |
|          |                                     | <b>3</b>  |  |

| Question                     | Answer   | Marks                        | Guidance   |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|------------------------------|--|------------------------------|--|--|-----------------|---------------|-----|-------------------|---------------|-----|---------------------|---------------|-----|---------------------|---------------|-----|---------------------|---------------|-----|-----------|--|
| 7(b)                         | <b>Method 1</b> Number of arrangements with A at each end – Number of arrangements with A at each end and 2 Ds together.   |                              |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | $\frac{7!}{2!} - 6!$   | <b>B1</b>                    | $\frac{7!}{2!} - e, {}^7P_5 - e, e$ a positive integer or 0.           |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              |  | <b>M1</b>                    | $d - \frac{6!}{r!}, d > 720, r = 1, 2.$                                |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | = 1800   | <b>A1</b>                    |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | <b>Method 2</b> A ^ ^ ^ ^ ^ A and Ds inserted separately   |                              |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | $5! \times \frac{{}^6P_2}{2!}$ or $5! \times \frac{6 \times 5}{2}$ or $5! \times {}^6C_2$  | <b>B1</b>                    | $5! \times s, s$ a positive integer, 1 may be implied.                 |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              |  | <b>M1</b>                    | $t \times \frac{6 \times 5}{u}, t$ a positive integer $> 1, u = 1, 2.$ |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | = 1800   | <b>A1</b>                    |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | <b>Method 3</b> Number of arrangements with As at each end and Ds placed in different scenarios.   |                              |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | <table border="1"> <thead> <tr> <th data-bbox="320 932 772 981">Scenario position of first D</th> <th data-bbox="772 932 896 981"></th> <th data-bbox="896 932 1008 981"></th> </tr> </thead> <tbody> <tr> <td data-bbox="320 981 772 1031">A D ^ ^ ^ ^ ^ A</td> <td data-bbox="772 981 896 1031"><math>5! \times 5</math></td> <td data-bbox="896 981 1008 1031">600</td> </tr> <tr> <td data-bbox="320 1031 772 1080">A ^ D ^ ^ ^ ^ ^ A</td> <td data-bbox="772 1031 896 1080"><math>5! \times 4</math></td> <td data-bbox="896 1031 1008 1080">480</td> </tr> <tr> <td data-bbox="320 1080 772 1129">A ^ ^ D ^ ^ ^ ^ ^ A</td> <td data-bbox="772 1080 896 1129"><math>5! \times 3</math></td> <td data-bbox="896 1080 1008 1129">360</td> </tr> <tr> <td data-bbox="320 1129 772 1179">A ^ ^ ^ D ^ ^ ^ ^ A</td> <td data-bbox="772 1129 896 1179"><math>5! \times 2</math></td> <td data-bbox="896 1129 1008 1179">240</td> </tr> <tr> <td data-bbox="320 1179 772 1228">A ^ ^ ^ ^ D ^ ^ ^ A</td> <td data-bbox="772 1179 896 1228"><math>5! \times 1</math></td> <td data-bbox="896 1179 1008 1228">120</td> </tr> </tbody> </table> | Scenario position of first D |  |  | A D ^ ^ ^ ^ ^ A | $5! \times 5$ | 600 | A ^ D ^ ^ ^ ^ ^ A | $5! \times 4$ | 480 | A ^ ^ D ^ ^ ^ ^ ^ A | $5! \times 3$ | 360 | A ^ ^ ^ D ^ ^ ^ ^ A | $5! \times 2$ | 240 | A ^ ^ ^ ^ D ^ ^ ^ A | $5! \times 1$ | 120 | <b>B1</b> | Correct outcome/value for 1 identified scenario, accept unsimplified, www. |
| Scenario position of first D |  |                              |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
| A D ^ ^ ^ ^ ^ A              | $5! \times 5$  | 600                          |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
| A ^ D ^ ^ ^ ^ ^ A            | $5! \times 4$  | 480                          |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
| A ^ ^ D ^ ^ ^ ^ ^ A          | $5! \times 3$  | 360                          |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
| A ^ ^ ^ D ^ ^ ^ ^ A          | $5! \times 2$  | 240                          |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
| A ^ ^ ^ ^ D ^ ^ ^ A          | $5! \times 1$  | 120                          |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              |  | <b>M1</b>                    | Add values of 5 correct scenarios, no incorrect/repeated scenarios.    |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              | [Total =] 1800   | <b>A1</b>                    |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |
|                              |  | <b>3</b>                     |  |  |                 |               |     |                   |               |     |                     |               |     |                     |               |     |                     |               |     |           |  |

| Question   | Answer  | Marks   | Guidance |  |         |   |      |         |   |      |           |  |
|--|---|---|----------|--|---------|---|------|---------|---|------|-----------|--|
| 7(c)   | <b>Method 1:</b>  |   |          |  |         |   |      |         |   |      |           |  |
|  | <table border="1"> <tr> <td data-bbox="315 280 517 347">Scenarios</td> <td data-bbox="517 280 775 347"></td> <td data-bbox="775 280 864 347"></td> </tr> <tr> <td data-bbox="315 347 517 414">A D ^ ^</td> <td data-bbox="517 347 775 414"><math>{}^2C_1 \times {}^2C_1 \times {}^5C_2</math></td> <td data-bbox="775 347 864 414">= 40</td> </tr> <tr> <td data-bbox="315 414 517 481">A D D ^</td> <td data-bbox="517 414 775 481"><math>{}^2C_1 \times [{}^2C_2 \times] {}^5C_1</math></td> <td data-bbox="775 414 864 481">= 10</td> </tr> </table> | Scenarios   |          |  | A D ^ ^ | ${}^2C_1 \times {}^2C_1 \times {}^5C_2$ | = 40 | A D D ^ | ${}^2C_1 \times [{}^2C_2 \times] {}^5C_1$ | = 10 | <b>M1</b> | At least one correct unsimplified expression for an identified scenario. |
|  | Scenarios   |   |          |  |         |   |      |         |   |      |           |  |
|  | A D ^ ^   | ${}^2C_1 \times {}^2C_1 \times {}^5C_2$   | = 40     |  |         |   |      |         |   |      |           |  |
|  | A D D ^   | ${}^2C_1 \times [{}^2C_2 \times] {}^5C_1$   | = 10     |  |         |   |      |         |   |      |           |  |
| [Total = ] 40 + 10 or 50 soi                     | <b>A1</b>   | www<br>If M0 scored, <b>SC B1</b> [total =]50 www.  |          |  |         |   |      |         |   |      |           |  |
| [Total number of selections =] ${}^9C_4$ [= 126] | <b>B1</b>   | Accept evaluated, accept as denominator of probability expression.<br>Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group.        |          |  |         |   |      |         |   |      |           |  |
| [Probability =] $\frac{50}{126}, \frac{25}{63}$  | <b>B1 FT</b>  | 0.396825... to at least 3SF.<br><br>FT $\frac{\textit{their attempted } 40 + 10}{126}$ . Numerator must be from an attempt to find the 2 appropriate scenarios and must be evaluated. |          |  |         |   |      |         |   |      |           |  |

| Question   | Answer           | Marks  | Guidance                            |           |   |
|--|------------------|--|-------------------------------------|-----------|---|
| 7(c)   | <b>Method 2:</b> |  |                                     |           |   |
|  | Scenarios        |  |                                     | <b>M1</b> | Numerator for at least one correct unsimplified expression for an identified scenario.<br>either $\frac{2 \times 2 \times 5 \times 4 \times 12}{a \times b \times c \times d}$ or $\frac{2 \times 2 \times 1 \times 5 \times 12}{a \times b \times c \times d}$<br>seen, $6 \leq a, b, c, d \leq 9$ . |
|  | A D ^ ^          | $\frac{2}{9} \times \frac{2}{8} \times \frac{5}{7} \times \frac{4}{6} \times {}^4P_2$            | $= \frac{960}{3024}, \frac{20}{63}$ |           | <b>A1</b>   |
|  | A D D ^          | $\frac{2}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{{}^4P_3}{2!}$ | $= \frac{240}{3024}, \frac{5}{63}$  | <b>B1</b> |   |
| [Total Probability =] $\frac{20}{63} + \frac{5}{63}$ |                  |  | <b>B1 FT</b>                        |           | 0.396825... to at least 3SF.<br><br>FT $\frac{\text{their attempted } 960 + 240}{3024}$ . Numerator must be from an attempt to find the 2 appropriate scenarios.  |
| $\frac{1200}{3024}, \frac{25}{63}$ oe                |                  |  |                                     |           |   |

| Question | Answer  | Marks        | Guidance   |
|----------|---|--------------|--|
| 7(c)     | <b>Method 3: selecting the A and then selecting 3 any letters and removing selections without Ds.</b> |              |  |
|          | ${}^2C_1 \times ({}^7C_3 - {}^5C_3) [= 2 \times (35 - 10)]$   | <b>M1</b>    | $a \times ({}^7C_3 - {}^5C_3), a = 1, 2.$  |
|          | [Total = ] 50   | <b>A1</b>    | www<br>If M0 scored, <b>SC B1</b> [total =]50 www.   |
|          | [Total number of selections =] ${}^9C_4 [= 126]$  | <b>B1</b>    | Accept evaluated, accept as denominator of probability expression.<br>Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group. |
|          | [Probability =] $\frac{50}{126}, \frac{25}{63}$   | <b>B1 FT</b> | 0.396825... to at least 3SF.<br>FT $\frac{\text{their attempted } 40 + 10}{126}$ . Numerator must be from an attempt to find the 2 appropriate scenarios.                      |
|          | <b>Method 4: Listing outcomes.</b>  |              |  |
|          | Either 10 correct outcomes for ADD^ listed or 40 correct outcomes for AD^^ listed                     | <b>M1</b>    |  |
|          | 50 stated   | <b>A1</b>    | www<br>If M0 scored, <b>SC B1</b> [total =]50 www.   |
|          | 126 stated or correct outcomes listed   | <b>B1</b>    |  |
|          | [Probability =] $\frac{50}{126}, \frac{25}{63}$   | <b>B1</b>    | 0.396825... to at least 3SF.<br>FT $\frac{\text{their attempted } 40 + 10}{126}$ . Numerator must be from an attempt to find the 2 appropriate scenarios.                      |
|          | <b>4</b>  |              |  |